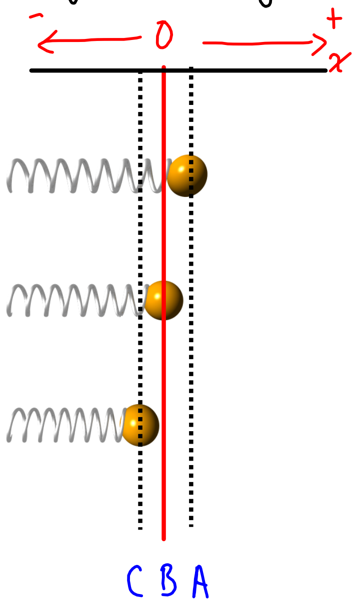
4.2 Energy Changes during SHM

Interchange of energy



at A → max displacement from equilibrium
→ max elastic potential energy
→ velocity is zero
→ Kinetic energy is zero

at B → at the equilibrium position
→ elastic potential energy is zer
→ velocity is a maximum
→ Kinetic energy is a maximum

at C → Same as A, but the displacement is negative.

There is a continuous exchange between elastic potential energy and kinetic energy.

Law of Conservation of Energy applies SHM (neglecting any damping forces).

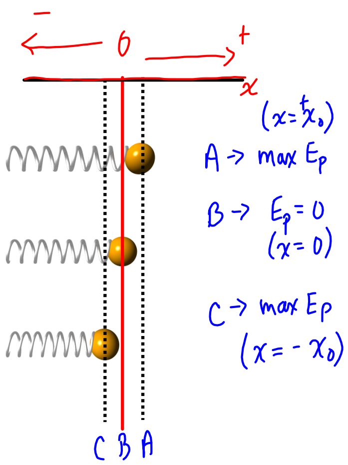
$$E_{total\ at\ A} = E_{total\ at\ B} = E_{total\ at\ C}$$

or anywhere in between.

The total mechanical energy is constant

$$E_T = E_k + E_p$$

Energy Analysis: Potential Energy



$$E_p = \frac{1}{2}kx^2$$

where k is the force constant

$$F_{net} = -kx$$

$$ma = -kx$$

$$a = \frac{-k}{m}x = -\omega^2 x$$

defining equation

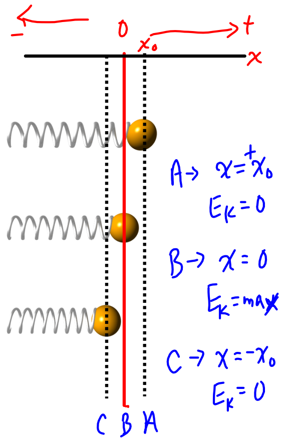
$$\therefore \quad \frac{k}{m} = \omega^2$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$k = m\omega^2$$

$$\therefore \quad \boxed{E_p = \frac{1}{2}\underbrace{m\omega^2}_{k}x^2}$$

Energy Analysis: Kinetic Energy



$$E_k = \frac{1}{2}mv^2$$

$$v^2 = \omega^2(x_0^2 - x^2)$$

$$\therefore \quad \boxed{E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)}$$

Energy Analysis: Total Energy

$$E_p = \frac{1}{2}m\omega^2 x^2$$

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2) \qquad \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x^2$$

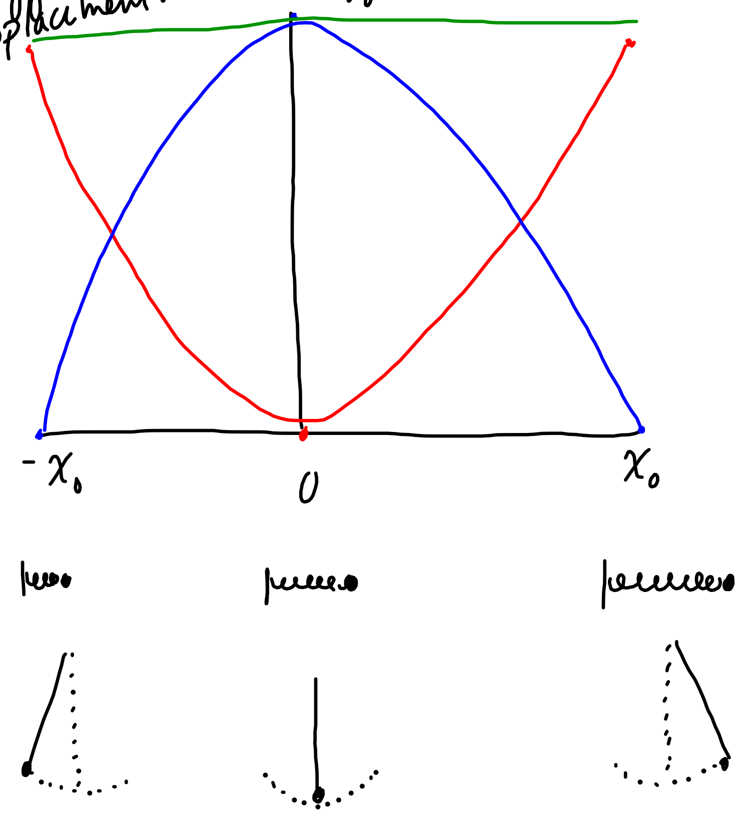
$$E_T = \frac{1}{2}m\omega^2 x_0^2$$

Total mechanical energy remains constant in the absence of damping forces

This total energy is essentially the elastic potential energy given to the mass-spring system by doing work.

Graphical Representation of Energy in SHM:

Energy vs Displacement:



$$E_p = \frac{1}{2} m \omega^2 x^2$$

$$E_k = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

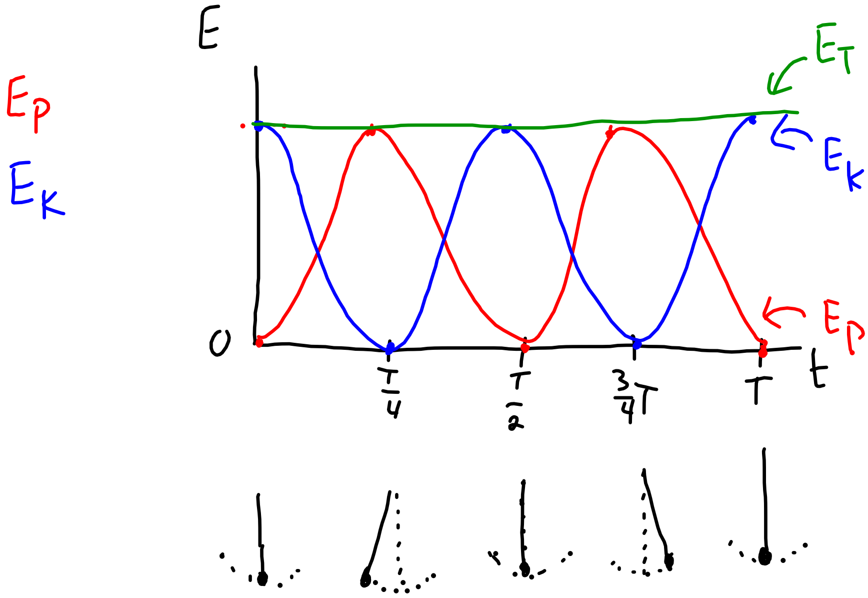
Graphical Representation of energy in SHM:

Energy versus time

Potential Energy: $E_p = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$

Kinetic Energy: $E_K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$

Total Energy: $E_T = \frac{1}{2} m \omega^2 x_0^2$ (constant!)



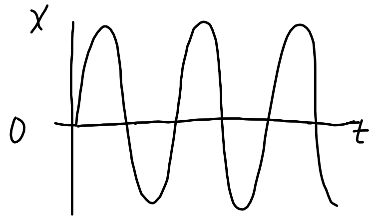
Data Booklet

$$E_K = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$$

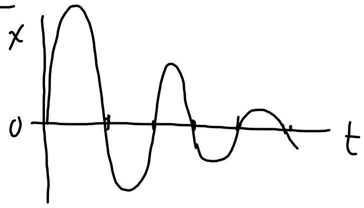
$$E_{K(max)} = \frac{1}{2} m\omega^2 x_0^2 \qquad (E_p = 0)$$

$$E_T = \frac{1}{2} m\omega^2 x_0^2$$

Undamped SHM



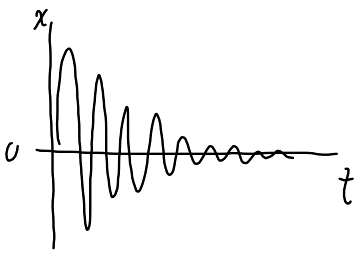
Damped



a resistive force (a dissipative force) causes this damping effect

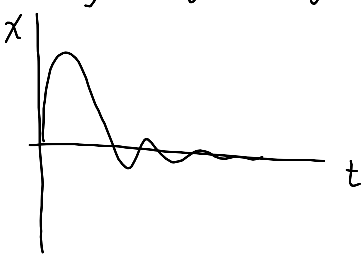
- the force opposes the oscillatory motion

Light damping (small damping force)



← longer time to decrease amplitude

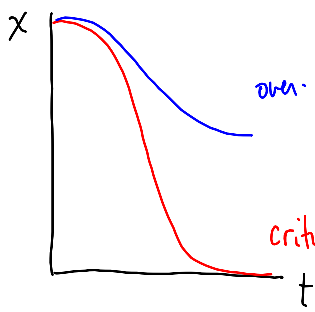
Heavy damping (large damping force)



← period is increased with an increase in the damping force.

The motion is still oscillatory.

Critical Damping + Over-Damping



over-damping (body returns to the equilibrium position, but takes longer)

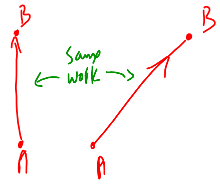
critical damping (the body returns to the equilibrium in the shortest time)

Dissipative Force

- work done depends on path taken

Friction is a dissipative force
- the work done against friction depends on the path.
- non-conservative force

Gravity is a non-dissipative force
- the path does not matter
- conservative force



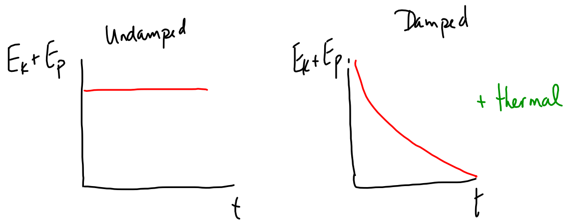
How does a dissipative force affect SHM?

Normal (undamped) SHM → continuous exchange of energy potential ↔ kinetic.
→ total mechanical energy remains the same.

Damped SHM (dissipative force) → continuous exchange of energy potential ↔ kinetic
→ total mechanical energy is continuously decreasing.
→ mechanical energy → thermal energy (degraded + cannot be returned to the body to keep the oscillation constant)

Dissipative Force causes:
1. continuous decrease in mechanical energy
2. lost energy is degraded energy

TOTAL ENERGY is still conserved!

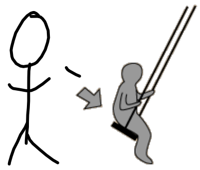


Examples of Damped Oscillation

- Damper on strings of Tennis Racket — critically damped to return to equilibrium position as soon as possible
- Vehicle suspension system — critical damping or heavy overdamping
- Ailerons on aircraft wings — critically damped to prevent catastrophic aileron flutter
- Building Design — reinforcement if structure to ensure that oscillation of the building is suitably damped
- Others? trampoline fabric. | magnetic damping in sensitive balances

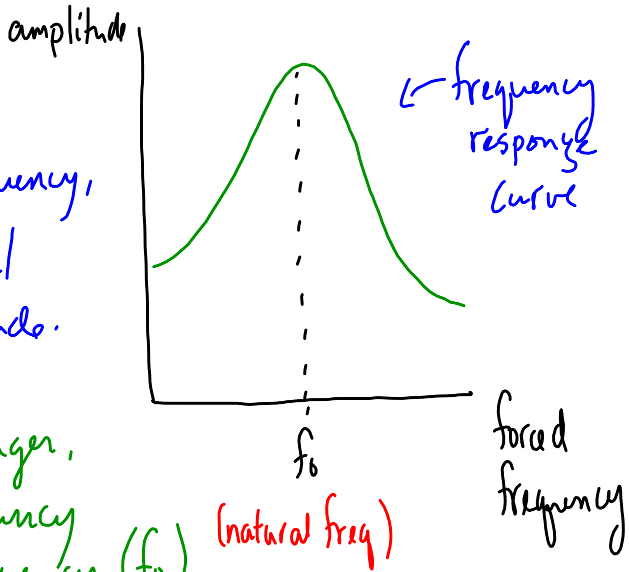
Consider swinging on a swing with no pumping or pushing, then you would swing with your natural frequency. (dissipates)

If your mum pushes you, then this is now a forced oscillation



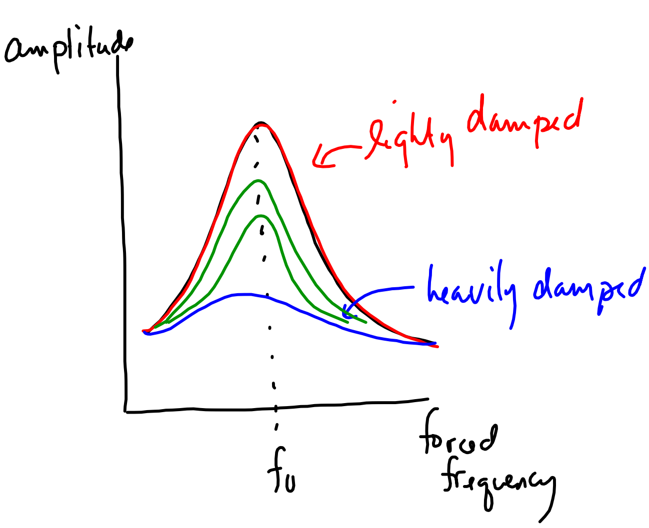
← oscillate with the frequency of the pushes

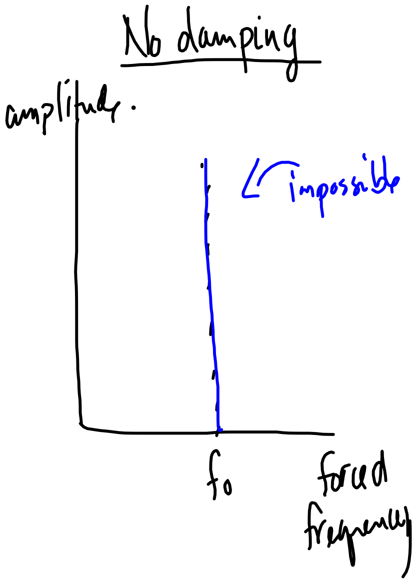
How is the amplitude of the oscillation affected by the frequency of the periodic impulses?



If the forced frequency matches the natural frequency, then the oscillations will have the greatest amplitude.

The amplitude gets larger, the closer the forced frequency is to the natural frequency (f_0)





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